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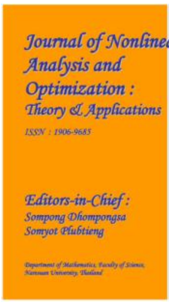

THE SEVEN BRIDGES OF KONIGSBERG-EULER'S SOLUTION

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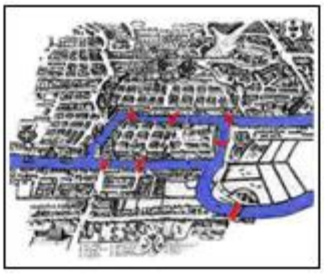
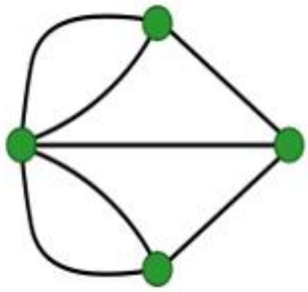
Abstract:
Euler states that if the sum of the number of times each letter must appear is one more then the total number of bridges, a journey can be made. However, if the number of occurrences is greater than one more than the number of bridges, a journey cannot be made, like the Königsberg Bridge problem. In modern terms, one replaces each land mass with an abstract "vertex" or node, and each bridge with an abstract connection, an "edge", which only serves to record which pair of vertices (land masses) is connected by that bridge. The resulting mathematical structure is a graph.

Definition of Euler circuit:
If there is a connected graph, which has a walk that passes through each and every edge of the graph only once, then that type of walk will be known as the Euler circuit.

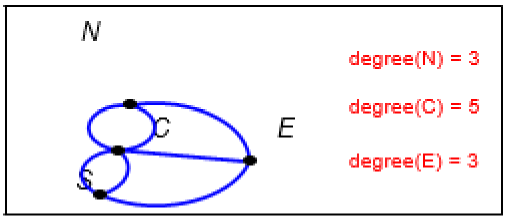
Euler's Analysis:
Euler first pointed out that the choice of route inside each land mass is irrelevant and that the only important feature of a route is the sequence of bridges crossed. This allowed him to reformulate the problem in abstract terms .Eliminating all features except the list of land masses and the bridges connecting them. In modern terms, one replaces each land mass with an abstract "vertex" or node, and each bridge with an abstract connection, an "edges", which only serves to record which pair of vertices (land masses) is connected by that bridge. The resulting mathematical structure is a graph.





Since only the connection information is relevant, the shape of pictorial representations of a graph may be distorted in any way, without changing the graph itself. Only the existence (or absence) of an edge between each pair of nodes is significant

Example: Königsberg:
Vertices=Regions of city
Degree of a vertex = Number of bridges that go to that region



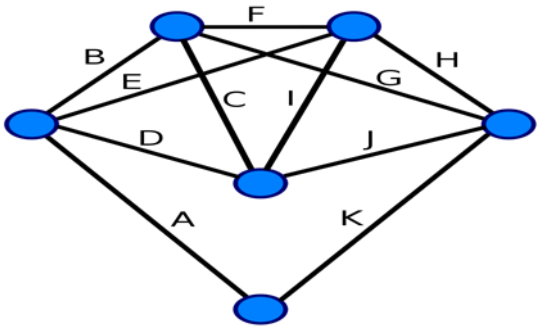
Definition of Eulerian:

An Eulerian trail or Euler walk, in an undirected is a walk that uses each edge exactly once. If such a walk exists, the graph is called traversable or semi-eulerian.

An Eulerian cycle also called an Eulerian circuit or Euler tour, in an undirected graph is a cycle that uses each edge exactly once. If such a cycle exists, the graph is called Eulerian or unicursal. The term "Eulerian graph" is also sometimes used in a weaker sense to denote a graph where every vertex has even degree. For finite connected graphs the two definitions are equivalent, while a possibly unconnected graph is Eulerian in the weaker sense if and only if each connected component has an Eulerian cycle.For directed graphs, "path" has to be replaced with directed path and "cycle" with directed cycle.

The definition and properties of Eulerian trails, cycles and graphs are valid for multigraph as well.

An Eulerian orientation of an undirected graph *G* is an assignment of a direction to each edge of *G* such that, at each vertex *v*, the in degree of *v* equals the outdegree of *v*. Such an orientation exists for any undirected graph in which every vertex has even degree, and may be found by constructing an Euler tour in each connected component of *G* and then orienting the edges according to the tour. Every Eulerian orientation of a connected graph is a strong orientation, an orientation that makes the resulting directed graph strongly connected.



Special Euler's properties :

(i) To get the Euler path a graph should have two or less number of odd vertices.
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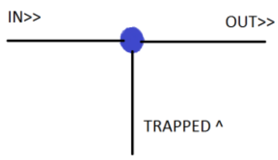
(ii) Starting and ending point on the graph is a odd vertex

Problem:

A vertex needs minimum of two edges to get in and out. If a vertex has odd edges then the person gets trapped. Hence every odd vertex should be a starting or ending point in the graph.



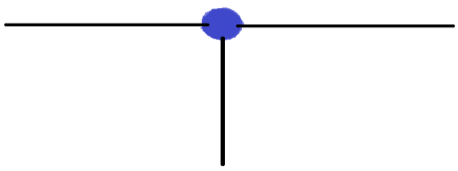
Odd and even vertex:

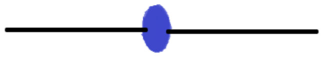
Odd vertex:

A vertex is called odd if it has an odd number of edges leading to it.

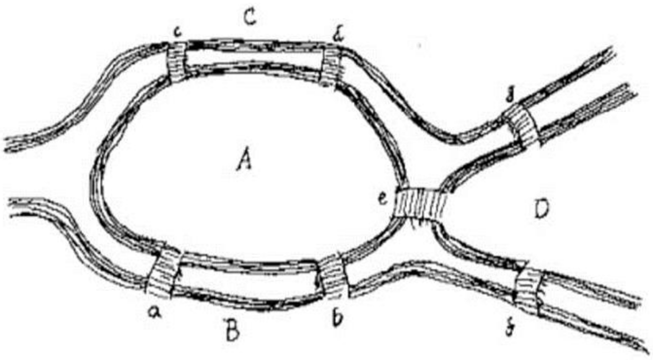
Even vertex:

A vertex is called even if it has an even number of edges leading to it.





Examples



Using the Konigsberg problem as his first example Euler shows the following:

Number of bridges = 7, Number of bridges plus one = 8

Region	Bridges	Times Region Must Appear
A	5	3
B	3	2
C	3	2
D	3	2

However, 3 + 2 + 2 + 2 = 9, which is more than 8, so the journey is impossible.Since this example is rather basic, Euler decides to design his own situation with two islands, four rivers, and fifteen bridges. Euler now attempts to figure out whether there is a path that allows someone to go over each bridge once and only once. Euler follows the same steps as above, naming the five different regions with capital letters, and creates a table to check it if is possible, like the following:

Number of bridges = 15, Number of bridges plus one = 16

Region	Bridges	Times Region Must Appear
A*	8	4
B*	4	2
C*	4	2
D	3	2
E	5	3
F*	6	3

In addition, 4 + 2 + 2 + 2 + 3 + 3 = 16, which equals the number of bridges, plus one, which means the journey is, in fact, possible. Since the sum equals the number of bridges plus one, the journey must start in either D or E. Now that Euler knows it is possible to make a journey, all he needs to do is state what the path will be. Euler chooses the path EaFbBcFdAeFfCgAhCiDkAmEnApBoElD, where he includes which bridges are crossed between the letters representing the landmasses. While this information is extraneous, as the exact bridge does not matter in knowing that a journey is possible, it is useful when selecting a path. This is a good example that shows the method which Euler would use when solving any problem of this nature.

Conclousion:

Therefore 3(for A) + 2(for B) + 2(for C) + 2(for D) = 9, but Euler already stated that there must only be eight occurrences for the seven bridges. This is a contradiction! Therefore, it is impossible to travel the bridges in the city of Königsberg once and only once.

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